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AN ANALYTICAL MODEL FOR PREDICTING GAIN, INTENSITY AND OUTPUT P--ETC(U)
FEB 82 G ZHI, Z SHUTAO

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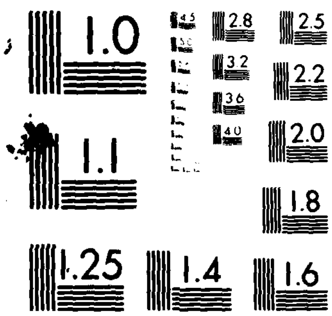

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AN ANALYTICAL MODEL FOR PREDICTING GAIN, INTENSITY AND OUTPUT POWER OF FAST SPEED FLOWING LASERS

by

Gao Zhi and Zhao Shutao

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AN ANALYTICAL MODEL FOR PREDICTING GAIN, INTENSITY AND OUTPUT POWER OF FAST SPEED FLOWING LASERS

by Gao Zhi and Zhao Shutao

(Institute of Mechanics, Academia Sinica)

Abstract

By phenomenal expression of the thermal motion distribution of gas particles based on the rate equations, ~~we have~~ deduced ~~the~~ analytical expressions for predicting gain, intensity and output power of gas flowing lasers, which are in agreement with the theoretical results given in article [1] for the homogeneous broadening limit. Present expressions may be reduced to the well-known formulae ~~[6]~~ of non-flowing gas lasers if flow velocity has vanished.

~~In the analysis~~, we ~~have considered~~ the effects of the changes in mirror reflectivity and the changes of the time-dependent gas excitation in upstream of the cavity.

I. Preface

The analytic expression of the CO_2 gas flowing laser is the solving of the rate equations [1,2] when the gain is equal to the loss and their results are applicable for homogeneous broadening. Article [3] analyzes the simultaneous action of homogeneous and heterogeneous broadening. Aside from this, when explaining the empirical results of the gas flowing laser, the semi-empirical analysis [4,5] of the gas flowing laser proposes the use of the theoretical formulae of the non-flowing gas

laser. However, from the theoretical results that already exist [1-3] for the gas laser, when flow velocity has vanished, the expression cannot be reduced to the corresponding relationship of non-flowing gas lasers [6].

By phenomenal expression of the thermal motion distribution based on the rate equations, we can deduce the theoretical relationship of the gain, intensity and output power of gas flowing lasers. When flow velocity has vanished, these relationships are reduced to the well-known formulae of non-flowing gas lasers. The results given in articles [1,2] are for the reducing of the homogeneous broadening limit. It should be pointed out that, strictly speaking, we cannot deduce the thermal motion distribution of gas particles from the rate equations, yet, from this, we can easily derive the desired results and provide a basis for further analysis.

II. Hypothesis and Basic Equations

Let us assume that the axis and flow directions are vertical, the gas flow passage is a rectangular section and the plane parallel mirror is placed on the two sides of the passage (see figure 1). The changes of the u,p and T gas flow parameters in the cavity as well as the effects of the boundary layer can be disregarded. The pump and cavity areas are separately opened in order to be able to analyze the "flow broadening" pulse width of the pulse laser in upstream of the cavity [7].

The working energy level of the CO_2-N_2system laser can be shown by five energy level models (see figure 2). Energy level 1 includes CO_2 symmetry and any energy level of the curved model. Because within the unit time the non-elastic collision causes the particle number of the i energy level to

j energy level to transport into $K_{ij}N_i$, N_i is the i energy level particle number density and the K_{ij} rate (unit seconds^{-1}) satisfies the relationship.

$$K_{32}, K_{21}, K_{10} \gg K_{31} > K_{0j}(j = 1, 2, 3) \qquad (2.1)$$

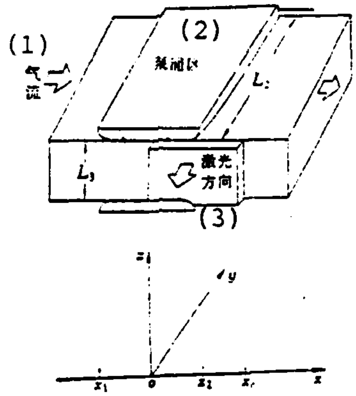

Fig. 1. Schematic Diagram and Coordinate System of Transverse Flow Gas Laser (Pump Area $x_1 \leqslant x \leqslant x_2$; $0 \leqslant x \leqslant x_c$=L)

1. Gas flow
2. Pump area
3. Laser direction

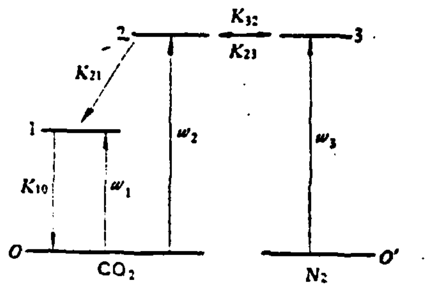

Fig. 2. The Working Energy Level and Relaxation Process of the CO_2-N_2 System Laser

When the gas flow parameters u,p and T do not change, the unsteady rate equation and radiation equation are:

$$
\left.
\begin{aligned}
&\frac{\partial N_3}{\partial t} + u\frac{\partial N_3}{\partial x} = w_3 - K_{32}N_3 + K_{23}N_2 \\
&\frac{\partial N_2}{\partial t} + u\frac{\partial N_2}{\partial x} = w_2 + K_{32}N_3 - (K_{21} + K_{23})N_2 - \frac{J}{ch\nu}(B'_{21}f_2N_2 - B'_{12}f_1N_1) \\
&\frac{\partial N_1}{\partial t} + u\frac{\partial N_1}{\partial x} = w_1 + K_{21}N_2 - K_{10}N + \frac{J}{ch\nu}(B'_{21}f_2N_2 - B'_{12}f_1N_1) \\
&\qquad N_2 + N_1 + N_0 = \text{常数}\ (1) \\
&\qquad\qquad N_{0'} + N_3 = \text{常数}\ (2)
\end{aligned}
\right\} \quad (2.2)
$$

1. Constant
2. Constant

$$\frac{\partial J}{\partial t} + c\,l\,\mathrm{grad}\,J = (B'_{21}f_2N_2 - B'_{12}f_1N_1)J \qquad (2.3)$$

In the formulas, t is the time, u is the flow velocity, w_i (i=1,2,3) is the pump rate, c is the speed of light, h is the Boltzman constant, v is the optical frequency, J is the radiation intensity, f_i (i=1,2) is the particle number fraction of the laser action in N_i, B'_{21} is the excited transmission rate when the Doppler apparent frequency is v' particles in a frequency of v radiation,

$$B'_{21} = B_{21}\frac{2/\pi\Delta\nu_N}{1+\left[\frac{2(\nu'-\nu)}{\Delta\nu_N}\right]^2}, \quad B'_{12} = \frac{g_2}{g_1}B'_{21} \tag{2.4}$$

g_i (i=1,2) is the energy level statistical weight, and Δv_N is the total width of the homogeneous broadening line's semi-peak value area. I is the light propagation direction. The intensity is sufficient for the following boundary conditions

$$\left.\begin{array}{l} y=0\ J_0^+ = R_1 J_0^-,\ R_1 = 1-a_1-t_1 \\ y=L_2\ J_{L_2}^- = R_2 J_{L_2}^+,\ R_2 = 1-a_2-t_2 \end{array}\right\} \tag{2.5}$$

R_i, a_i and t_i (i=1,2) are separately the reflection, absorption and transmission rates of the two ends of the mirror, J^+ and J^- are the advancing photon energy flows along the y positive and negative directions, and $J=J^++J^-$.

III. Solution

In the two ends of formula (2.3), aside from the $\frac{\partial J}{\partial y}$, all of the quantities are small and thus can be omitted. Therefore, from formulas (2.3) and (2.5) we obtain:

$$g = \frac{1}{L_2}\int_0^{L_2} \frac{1}{c}(B'_{21}f_2N_2 - B'_{12}f_1N_1)dy = \frac{-1}{2L_2}\ln R_1R_2 \tag{3.1}$$

Because [1] $J_{max}/J_{min}=2\sqrt{R}/(1+R)$ and $R=\min(R_1,R_2)$, when $R \lesssim 0.6$, the changes of J along the y direction can be disregarded and therefore from formulas (2.3) and (2.5) we obtain:

$$gI = \frac{1}{L_2}\int_0^{L_2} \frac{1}{c}(B'_{21}f_2N_2 - B'_{12}f_1N_1)Jdy = \frac{1}{L_2}(J^+_L - J^-_L + J^-_0 - J^+_0) \tag{3.2}$$

$$J_0 = J^-_0(1+R_1) = \frac{gIL_2(1+R_1)\sqrt{R_2}}{(\sqrt{R_1}+\sqrt{R_2})(1-\sqrt{R_1R_2})},\quad I = \frac{1}{L_2}\int_0^{L_2} Jdy \tag{3.3}$$

Here, g takes the average gain coefficient for y and formula (3.1) is then the threshold value conditions of the gas flow laser vibration. Generally speaking, g changes in accordance with x.

Below, in order to find the solution for the rate equation, we take the integral mean of formula (2.1) for y and carry out the following mathematical transformation

$$\xi = \frac{x}{u},\quad \zeta = t - \frac{x}{u} \tag{3.4}$$

The transformation of formula (2.1) is:

$$
\left.
\begin{aligned}
&\frac{\partial^2 n_b}{\partial \xi^2} + (K_{32} + S_1 K_{23} + S_2 K_{10}) \frac{\partial n_b}{\partial \xi} + S_2 K_{32} K_{10} n_b \\
&\qquad = K_{32} \sum_{i=1}^{2} w_i + \frac{\partial (w_1 + w_2)}{\partial \xi} + S_0 \left[(K_{10} - K_{23}) \frac{\partial g}{\partial \xi} + K_{32} K_{10} g \right] \\
&\frac{\partial^2 n_3}{\partial \xi^2} + (K_{32} + S_1 K_{23} + S_2 K_{10}) \frac{\partial n_3}{\partial \xi} + S_2 K_{32} K_{10} n_3 \\
&\qquad = S_1 K_{23} \sum_{i=1}^{3} w_i + S_2 K_{10} w_3 + \frac{\partial w_3}{\partial \xi} + S_0 K_{23} \left(\frac{\partial g}{\partial \xi} + K_{10} g \right)
\end{aligned}
\right\} \quad (3.5)
$$

In the formulas

$$
n_b = n_1 + n_2, \quad n_2 = S_1 n_b + S_0 g, \quad n_1 = S_2 n_b - S_0 g \qquad (3.6)
$$

$$
S_0 = \frac{c}{B'_{21} j_2 + B'_{12} j_1}, \quad S_2 = \frac{B'_{21} j_2}{B'_{21} j_2 + B'_{12} j_1}, \quad S_1 + S_2 = 1 \qquad (3.7)
$$

$$
n_i = \frac{1}{L_2} \int_0^{L_2} N_i \, dy
$$

We took $g = g(\zeta) e^{\delta \xi}$ and found the solution for formula (3.5)

$$
\left.
\begin{aligned}
n_b &= n_{bp} + S_0 g n_{bg} + \sum_{\substack{i=1 (j \neq i, \\ j = 1, 2)}}^{2} \frac{e^{-\lambda_i \xi}}{\lambda_i - \lambda_j} \{ (w_1 + w_2 - \lambda_j n_{bp}) + K_{32} n_3^0(\zeta) \\
&\quad + (\lambda_j - S_1 K_{23} - S_2 K_{10}) n_b^0(\zeta) + S_0 g [(K_{10} - K_{23}) - (\lambda_j + \delta) n_{bg}] e^{-\delta \xi} \} \\
n_3 &= n_{3p} + S_0 g n_{3g} + \sum_{\substack{i=1 (j \neq i, \\ j = 1, 2)}}^{2} \frac{e^{-\lambda_j \xi}}{\lambda_j - \lambda_i} \{ (w_3 - \lambda_j n_{3p}) + (\lambda_j - K_{32}) n_3^0(\zeta) \\
&\quad + S_1 K_{23} n_b^0(\zeta) + S_0 g [K_{23} - (\lambda_j + \delta) n_{3g}] e^{-\delta \xi} \}
\end{aligned}
\right\} \quad (3.8)
$$

In the formulas, the zero in the upper right indicates the distribution of the corresponding quantities in the $\xi = 0$ (that is, $x = 0$) area, δ is a constant, and w_i is a constant.

$$\lambda_{1,2} = \frac{1}{2}\left[(K_{32} + S_1K_{23} + S_2K_{10}) \pm \sqrt{(K_{32} + S_1K_{23} + S_2K_{10})^2 - 4S_2K_{32}K_{10}}\right]$$

$$n_{bg} = \frac{(K_{10} - K_{23})\delta + K_{32}K_{10}}{\delta^2 + (K_{32} + S_1K_{23} + S_2K_{10})\delta + S_2K_{32}K_{10}},$$

$$n_{3g} = \frac{K_{23}(K_{10} + \delta)}{\delta^2 + (K_{32} + S_1K_{23} + S_2K_{10})\delta + S_2K_{32}K_{10}}$$

$$n_{br} = \begin{cases} \dfrac{1}{S_2K_{10}} \sum\limits_{i=1}^{3} w_i & 0 \leqslant \xi \leqslant \xi_2 = \dfrac{x_2}{u} > 0 \\ 0 & \xi > \xi_2 \end{cases}$$

$$n_{3r} = \begin{cases} \dfrac{S_2K_{10}w_3 + S_1K_{23} \sum\limits_{i=1}^{3} w_i}{S_2K_{32}K_{10}} & 0 \leqslant \xi \leqslant \xi_2 \\ 0 & \xi > \xi_2 \end{cases}$$

The solution of formula (3.8) is $\lambda_1 = \lambda_2$ and the solution of $\lambda_1 \neq \lambda_2$ can be found in the same way. We will not discuss this here. The explanation of the initial distribution of x=0 is as follows: 1. the solution of the initial distribution is given from the non-radiative $J \equiv 0$ of formula (2.2). We will not write the non-radiative solution here. When the continuous pump is in upstream of the cavity, $n_i^0(\zeta)|_{\xi=0} = n_i^0(t)|_{x=0}$ = a constant, and when a pulse pump, $n_i^0(t)|_{x=0}$ changes in accordance with t. 2. The non-radiative solution is generally not sufficient for formula (3.1) but because $\frac{2L_2}{c(1-R_1R_2)} \ll K_{32}^{-1}, K_{10}^{-1}$ and $\frac{n_0}{w_i}$, therefore the colliding non-elastic transformation does not have enough time in the $\frac{2L_2}{c(1-R_1R_2)}$ time range. Moreover, $\frac{2L_2}{c(1-R_1R_2)} \ll L_2$ and therefore when $\xi = 0$

$$n_b^0 = n_2^0(\zeta) + n_1^0(\zeta), \quad n_3^0 = n_3^0(\zeta) \tag{3.9}$$

In the equation, $n_i^0(\zeta) = n_i^0(t)|_{x=0}$ (i=1,2,3) is obtained from the non-radiative solution. 3. Usually, $K_{32}, K_{10} \gg K_{21}, \frac{u}{L_1}$ and

therefore $K_{32}n_3^0(\zeta) = K_{23}n_2^0(\zeta)$.

We obtain gI from formulas (2.2) and (3.6):

$$
\begin{aligned}
\frac{gI}{h\nu} &= S_2 w_2 - S_1 w_1 + S_2 K_{32} n_3 - S_1 f n_b - S_0 g(\delta + K_{21} + S_2 K_{23} + S_1 K_{10}) \\
&= \frac{2}{\pi \Delta\nu_N} \frac{K_0 I_s}{h\nu} \exp\left\{-\left[\frac{2(\nu' - \nu_0)\sqrt{\ln 2}}{\Delta\nu_D}\right]^2\right\} \\
&- \left(1 + \left[\frac{2(\nu' - \nu)}{\Delta\nu_N}\right]^2\right) \frac{gI_s}{h\nu}
\end{aligned}
\tag{3.10}
$$

In the formula

$$
f = K_{21} + S_2 K_{23} - S_2 K_{10} \tag*{(3.10)$_1$}
$$

$$
\begin{aligned}
&\frac{2}{\pi \Delta\nu_N} \frac{K_0 I_s}{h\nu} = \left(w_{20} + w_{30} - \frac{S_1 K_{23}}{S_2 K_{10}} \sum_{i=1}^{3} w_{i0}\right) + \sum_{\substack{i=1 (j \neq i \\ j=1,2)}}^{2} \frac{e^{-\lambda_i \xi}}{\lambda_j - \lambda_i} \{S_2 K_{32}(w_{30} - \lambda_i n_{3p}) \\
&- S_1 f(w_{10} + w_{20} - \lambda_i n_{bp}) + [S_2 K_{32}(\lambda_i - K_{32}) - S_1 f K_{32}] n_{30}^0(\zeta) \\
&+ [S_1 S_2 K_{23} K_{32} - S_1 f(\lambda_i - S_1 K_{23} - S_2 K_{10})] n_{b0}^0(\zeta)\} \\
&\frac{I_s}{h\nu} = \frac{\pi \Delta\nu_N}{2} \frac{c S_2}{B_{21} f_2} \Big\{(K_{21} + S_2 K_{23} + S_1 K_{10} + \delta - S_2 K_{32} n_{3g} + S_1 f n_{bg}) \\
&- \sum_{\substack{i=1 (j \neq i, \\ j=1,2)}}^{2} \frac{e^{-(\lambda_i + \delta)\xi}}{\lambda_j - \lambda_i} (S_2 K_{32}[K_{23} - (\lambda_i + \delta) n_{3g}] - S_1 f[(K_{10} - K_{23}) \\
&- (\lambda_i + \delta) n_{bg}])\Big\}
\end{aligned}
$$

When deriving formula (3.10), we had already assumed that the plane motion is in a quasi-equilibrium and the local Maxwell rate distribution is established, that is

$$w_i = w_{i0}\exp\left\{-\left[\frac{2(\nu'-\nu_0)\sqrt{\ln 2}}{\Delta\nu_D}\right]^2\right\},$$
$$n_i = n_{i0}\exp\left\{-\left[\frac{2(\nu'-\nu_0)\sqrt{\ln 2}}{\Delta\nu_D}\right]^2\right\}$$

v_o is the Doppler line central frequency and Δv_D is the total width of the Doppler line semi-peak value area.

IV. Gain, Intensity and Output Power

Using formula (3.10) for the identical optical frequency and Doppler central frequency which is $v=v_o$, we obtain

$$g = \frac{K_0}{\pi}\int_{-\infty}^{\infty}\frac{\exp(-z^2\eta^2)}{1+z^2+I/I_s}dz = \frac{K_0\exp\left[\eta^2\left(1+\frac{I}{I_s}\right)\right]}{\sqrt{1+I/I_s}}\left\{1-\operatorname{erf}\left(\eta\sqrt{1+\frac{I}{I_s}}\right)\right\} \quad (4.1)$$

In the formula, $\eta = \frac{\Delta v_N}{\Delta v_D}\sqrt{\text{In}2}$, $z=\frac{2(v'-v_o)}{\Delta v_N}$, and erf is the probability integral. When there is heterogeneous broadening, then $\eta \to 0$ and formula (4.1) changes to

$$g = \frac{K_0(\xi,\zeta)}{\sqrt{1+I/I_s(\xi)}} \quad (4.2)$$

When there is homogeneous broadening, then $\eta \to \infty$ and formula (4.1) changes to

$$g = \frac{g_0(\xi,\zeta)}{1+I/I_s(\xi)}, \quad g_0 = \frac{K_0}{\eta\sqrt{\pi}} \quad (4.3)$$

I_s is defined as the local saturation intensity, K_0 and g_0 are separately the heterogeneous and homogeneous broadening unsaturated gain coefficients. If the above formulae are applied when the gain is equal to the loss, they are identical in form to the well-known formulae [6] of non-flowing gas lasers. When $\lambda_i \xi \gg 1$,

$$I_s \rightarrow \bar{I}_s = \frac{\pi \Delta \nu_N}{2} \frac{ch\nu S_2}{B_{21} f_2} \frac{K_{21} K_{32} K_{10} + \delta [K_{21} K_{32} + (K_{32} + K_{2r} + K_{21}) K_{10}]}{S_2 K_{32} K_{10} + (K_{32} + S_1 K_{21} + S_2 K_{10}) \delta} \qquad (4.4)$$

$$K_0 I_s \rightarrow \bar{K}_0 \bar{I}_s = \frac{\pi \Delta \nu_N h\nu}{2} \left(w_{20} + w_{30} - \frac{S_1 K_{21}}{S_2 K_{10}} \sum_{i=0}^{3} w_{i0} \right)$$

In formula (4.4), the depleted δ^2 is a high order number. It is easy to obtain the radiation intensity in the cavity from formulas (4.1)-(4.3). Transmission radiation intensity J_t is

$$J_t = J_0^- t_1 + J_L^+ t_2 = \frac{g I L_2 (t_1 \sqrt{R_2} + t_2 \sqrt{R_1})}{(\sqrt{R_1} + \sqrt{R_2})(1 - \sqrt{R_1 R_2})} \qquad (4.5)$$

When J_t is opposite the x and t integrals, we obtain the output power, and when it is opposite one end putting out to another end the reflector has no loss so that when R_2=1 we obtain:

$$P = \frac{V}{L_1(\zeta - \zeta_0)} \int_{\zeta_0}^{\zeta} \int_0^{t_c} \frac{t_1 u}{a_1 + t_1} \frac{K_0 I \exp\left[\eta^2 \left(1 + \frac{I}{I_s}\right)\right]}{\sqrt{1 + I/I_s}} \left[1 - \mathrm{erf}\left(\eta \sqrt{1 + \frac{I}{I_s}}\right)\right] d\xi d\zeta \Big|_{\xi=0} \qquad (4.6)$$

In the formula, $V=L_1 L_2 L_3$ is the cavity volume. When $\eta \gg 1$, formula (4.6) changes to

$$P = \frac{uVt_1}{L_1(\zeta-\zeta_0)(a_1+t_1)}\left\{\int_{\zeta_0}^{\zeta}\int_0^{\xi_c} g_0 I_s d\xi d\zeta \Big|_{\xi=0} + \frac{\ln R_1^0}{L_2}\int_{\zeta_0}^{\zeta}\int_0^{\xi_c} I_s e^{\delta\xi} d\xi d\zeta \Big|_{\xi=0}\right\}$$
$$= \frac{t_1 V I_s^*}{L_2(a_1+t_1)}(g_0^* L_2 + \ln R_1^0) \quad (4.7)$$

In the formula

$$I_s^* = \frac{u}{L_1(\zeta-\zeta_0)}\int_{\zeta_0}^{\zeta}\int_0^{\xi_c} I_s e^{\delta\xi} d\xi d\zeta \Big|_{\xi=0},$$
$$g_0^* L_2 = \frac{L_2\int_{\zeta_0}^{\zeta}\int_0^{\xi_c} g_0 I_s d\xi d\zeta \Big|_{\xi=0}}{\int_{\zeta_0}^{\zeta}\int_0^{\xi_c} I_s e^{\delta\xi} d\xi d\zeta \Big|_{\xi=0}} \quad (4.8)$$

I_s^* and g_o^* can be taken as the mean saturation intensity and mean unsaturated gain coefficients. They are taken as the means of space and time. Formula (4.7) is identical in form to the Rigrod formula [6] of the non-flowing gas laser. When formula (3.10) is substituted into formula (4.8), we can obtain I_s^* and g_o^*,

$$I_s^* = \frac{\pi\Delta\nu_N}{2}\frac{ch\nu u S_2}{B_{21} f_2 L_1}\Big\{(K_{21} + S_2K_{23} + S_1K_{10} + \delta - S_2K_{32}n_{3g} + S_1 f n_{bg})\frac{e^{\delta L_1/u}-1}{\delta}$$
$$- \sum_{\substack{i=1 (j\neq i \\ j=1,2)}}^{2} \frac{1-e^{-\lambda_i L_1/u}}{\lambda_i(\lambda_j-\lambda_i)}(S_2K_{32}[K_{32} - (\lambda_i+\delta)n_{3g}]$$
$$- S_1 f[(K_{10} - K_{23}) - (\lambda_i+\delta)n_{bg}])\Big\} \quad (4.9)$$

When the g_o^* expression is omitted, the output power's maximum and optimum output coupling t_i^* is found based on the relationship

of $\frac{\partial P}{\partial t_1}=0$, and when $\delta=0$, we obtain

$$\frac{t_1^*}{a_1} = \frac{1 - a_1 - t_1^*}{a_1 + t_1^*}\left[g_0^* L_2 + \ln(1 - a_1 - t_1^*)\right] \qquad (4.10)$$

We found the maximum output power P^* from formulas (4.7) and (4.10):

$$P^* = \frac{V I_s^* (t_1^*)^2}{L_2 a_1 (1 - a_1 - t_1)} = \frac{(t_1^*)^2}{a_1(1 - a_1 - t_1)} \frac{uV}{L_1 L_2 (\zeta - \zeta_0)} \int_{\zeta_0}^{\zeta} \int_0^{\xi_c} I_s \, d\xi \, d\zeta \Big|_{\xi=\infty} \qquad (4.11)$$

The above formula is identical to the Rigrod formula [6]. In the same way, we can discuss the formula changes for $\eta \ll 1$ heterogeneous broadening.

V. Parametric Analysis

I_s forms a direct ratio with pressure square p^2 but has no relation to the gas excitation of the pump rate and cavity entrance. It can be known from formula (3.10) that I_s declines in accordance with $\frac{x}{u}$ monotone and from the maximum value $I_{s,max}$ of x=0 it tends toward $\bar{I}_s$ (see formula (4.4))

$$I_{s,max} = I_s|_{x=0} = (K_{21} + S_2 K_{23} + S_1 K_{10} + \delta) \frac{ch\nu S_2}{B_{21} f_2} \frac{\pi \Delta \nu_V}{2} \qquad (5.1)$$

When we raise the flow velocity (other parameters are fixed), $I_{s,max}$ does not change yet I_s and $\bar{I}_s$ noticeably increase (see later section on calculation examples). The influence of the

reflectivity $R_1=(R_1^0)^{2\delta\xi}$ changes on I_s are great and the rational range of δ is deduced as:

$$|\delta| < \min(\lambda_1, \lambda_2) \overset{(1)}{\text{即}} |\delta| = O(K_{21}) \tag{5.2}$$

1. Which is

It can be known from formula (4.4) that $\delta \approx -\frac{K_{21}K_{32}}{K_{32}+K_{23}}$ and $\bar{I}_s \approx 0$. This is because when $\delta < 0$, R_1 increases in accordance with the $\frac{x}{u}$ monotone and decreases in accordance with the t_1 monotone. When t_1 decreases to zero, light emission is forced to stop; when $\delta > 0$, $\bar{I}_s|_{\delta>0}$ (or $\gg$) $\bar{I}_s|_{\delta=0}$.

In the gas flowing laser, the I_s is still related to the K_o and g_o parameters and the mirror surface conditions. Only when u=0 and δ =0 do these parameters possess the physical implications of corresponding parameters in non-flowing steady state gas lasers. This explains the non-steady state quality of the gas laser. The parameter of I_s^* depends on its similarity to I_s.

If pump rate w_i forms a direct ratio with p, then the unsaturated gain coefficient K_o has no relation to p, and g_o and p form a direct ratio. K_o and g_o form a direct ratio with w_i and the gas excitation of the cavity entrance area. It can be shown from formulas (3.10) and (4.4) that when the pump is in upstream of the cavity, $\delta \geqslant 0$ and the g_o monotone declines; when $\delta < 0$, the g_o monotone rises. When the pump is in the cavity, we obtain from formula (4.4):

$$\bar{K}_0 = \left(w_{20} + w_{30} - \frac{S_1 K_{21}}{S_2 K_{10}} \sum_{i=1}^{3} w_{i0}\right) \cdot \frac{B_{21} f_2 [S_3 K_{32} K_{10} + (K_{12} + S_1 K_{23} + S_2 K_{10})\delta]}{c S_2 \{K_{21} K_{32} K_{10} + [K_{21} K_{32} + (K_{32} + K_{23} + K_{21}) K_{10}]\delta\}} \quad (5.3)$$

Therefore, when pump conditions are similar, $\bar{K}_0|_{\delta>0} <$ (or $\ll$) $\bar{K}_0|_{\delta=0}$.

Output power P forms a direct ratio with the p and u pump speed and the gas excitation of the cavity entrance area. When reflectivity is an unchanging $\delta=0$, the maximum output power P^* forms a direct ratio with p^2 and the mirror area.

VI. Calculation Examples

See the following table [8] which uses a rate constant for the calculation of a $CO_2/N_2/He$ gas compound.

T=300K Rate Constant Value

N_{He}/N	(1) K_{10}/p (托$^{-1}$秒$^{-1}$)	(2) K_{21}/p (托$^{-1}$秒$^{-1}$)	(3) K_{23}/p (托$^{-1}$秒$^{-1}$)	K_{32}/K_{23}
0	8.6×10^{2}	1.23×10^{5}	1.67×10^{4}	
0.3	1.2×10^{3}	1.03×10^{5}	1.16×10^{4}	N_{CO_2}/N_{N_2}
0.5	1.9×10^{3}	9.6×10^{4}	8.30×10^{3}	

1. (Torr^{-1}seconds^{-1})
2. (Torr^{-1}seconds^{-1})
3. (Torr^{-1}seconds^{-1})

From the data in article [8] we derive

$$\frac{S_2}{S_0} = \frac{\lambda_2^2\theta_r}{4\pi\tau_{21}\nu_c}\frac{2(2l+1)}{T} \cdot \exp\left[-l(l+1)\frac{\theta_r}{T}\right] \approx \frac{718}{NT}\ (\text{厘米}^2) \quad (1)$$

1. (Centimeters^2)

In the same way, S_1/S_2 is the function of the gas temperature and excitation. When T=300K and excitation is not high, $\frac{S_1}{S_2} \approx 0.04$.

I_s changes in accordance with $\frac{\lambda_2 x}{u}$ ($\lambda_2 < \lambda_1$) (see figure 3), is in the range of 10^3-10^1 watts/centimeter^2, and is in agreement with the test results [9]. The calculation of the I_s curve confirmed the last sections analytical conclusions. When $\delta = \frac{\lambda_2}{5}$, I_s is about ten times greater than $I_s|_{\delta=0}$.

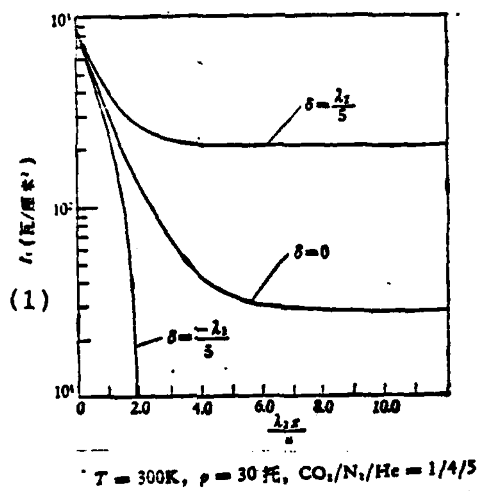

Fig. 3. I_s Changes in Accordance With $\frac{\lambda_2 x}{u}$

1. I_s (watts/centimeter^2)
2. Torr

The g_o and I change in accordance with $\frac{\lambda_2 x}{u}$ and $\frac{ut}{L_1}$ (see figures 4 and 5). The time changes of the pump pulse in upstream of the cavity are indicated by n_2^o which changes in accordance with $\frac{ut}{L_1}$. The g_o and I seem like a "wave" advancing toward the right. The straight lines $\frac{tu}{L_1} - b\,\frac{\lambda_2 x}{u} = 0$ and 1 are the front and back surfaces. Here, $b = \frac{u}{\lambda_2 L_1}$. It can be shown from figures 4 and 5 that along the straight line $\frac{tu}{L_1} - b\,\frac{\lambda_2 x}{u} = H =$ a constant, $0 < H < 1$ and the changes of g_o are slower than those of I.

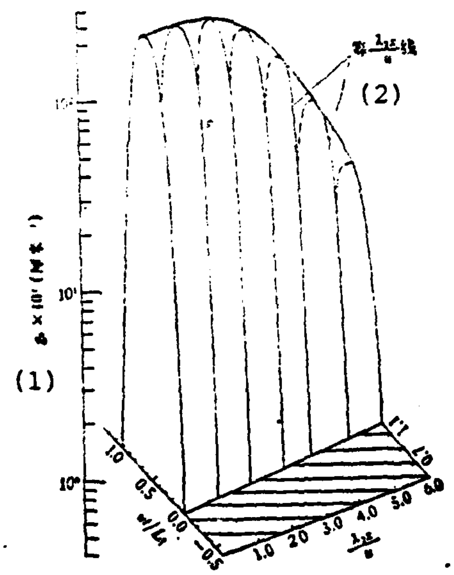

Fig. 4. g_o Changes in Accordance With $\frac{\lambda_2 x}{u}$ and $\frac{ut}{L_1}$

1. $g_o \times 10^4$ (centimeter^{-1})
2. Equal $\frac{\lambda_2 x}{u}$ lines

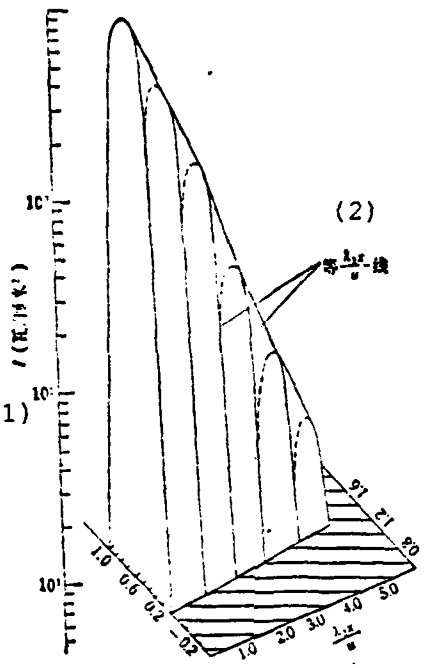

Fig. 5. I Changes in Accordance With $\frac{\lambda 2^x}{u}$ and $\frac{ut}{L_1}$

1. I (watts/centimeter^2)
2. Equal $\frac{2^x}{u}$ lines

For the continuous pump ~~is~~ upstream of the cavity, when R_7= a constant, $\bar{g}_0^*$ slowly declines in ~~accordance~~ with the $\frac{\lambda_2 x}{u}$ monotone, $\frac{\lambda_2 x}{u}$=10 and $\bar{g}_o^*$ is about 0.8; when $\delta = \frac{\lambda_2}{5}$, the $\bar{g}_0^*$ monotone decreases and rather marvedly, $\frac{\lambda_2 x}{u}$ =10 $\bar{g}_0^*$ is approx. 0.1 when $\delta = \frac{-\lambda_2}{5}$ $\bar{g}_0^*$ rises with $\frac{\lambda_2 x}{u}$ and I_s decreases

with $\frac{\lambda_2 x}{u}$. It should be noted that neither $\bar{g}_o^*$ or I_s^* have any noticeable physical significance. It can be known from figures 4 and 6 that when R is a constant whether it is a continuous or pulse pump in upstream of the cavity, g_o and g_o^* both change slowly in accordance with $\frac{x}{u}$. This is a certain mean gain coefficient selected from articles [5] and [7] which provides physical data for using the non-flowing formula to calculate the gas flowing laser output power.

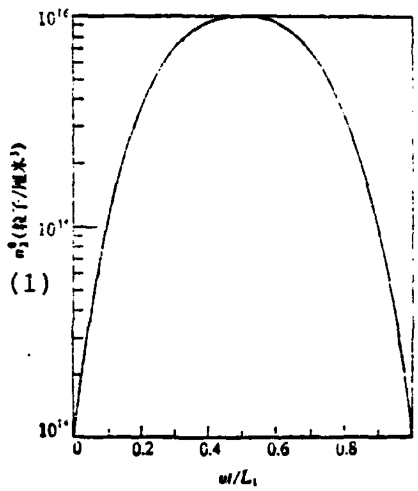

Fig. 4.1. The Changes of Cavity Entrance n_2^o in Accordance With $\frac{ut}{L_1}$

1. n_2^o (particles/centimeter^3)

T=300K, p=30 torr, $\delta=0$, $g_{x=0}=2\times10^{-3}$ centimeter^{-1}, $CO_2/N_2/He=1/4/5$

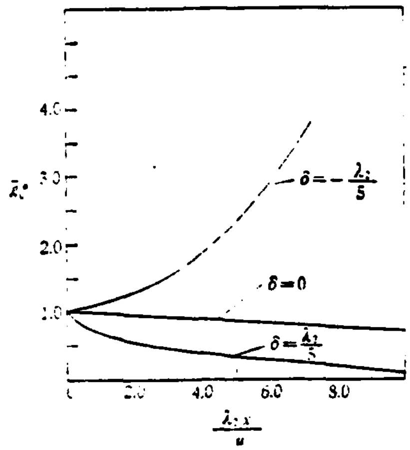

Fig. 6. The Normalized Mean Unsaturated Gain $\bar{g}_o^*$ Changes in Accordance With $\frac{\lambda_2 x}{u}$

T=300K, p=30 torr, $CO_2/N_2/He=1/4/5$

The calculation of the gas flowing laser output power is in agreement with the curve calculations of article [1] (see figure 7). The numerical value of $\frac{\lambda_2 x}{u}$ indicates the level of the effective vibration which is able to be extracted. When $\frac{\lambda_2 x}{u} \approx 5$, the effective vibrations can be completely extracted for the calculation parameters of figure 7.

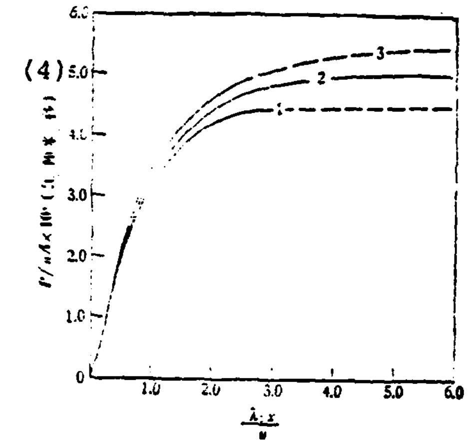

Fig. 7. Gas Flowing Laser Output Power $\frac{P}{uA}$ Changes in Accordance With $\frac{\lambda_2 x}{u}$

1. $\delta = \frac{\lambda_2}{5}$, 2. $\delta = 0$, 3. $\delta = -\frac{\lambda_2}{5}$, T=300K, p=30 torr, $CO_2/N_2/He=1/4/5$, $\frac{t_1}{a_1+t_1}=0.7$, $g_{x-0}=2\times10^{-3}$ centimeter^{-1}, $n_2^o=10^{16}$ particles/centimeter^3

4. (watts/centimeter^2/seconds)

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